Cubic Stress Tensor Sensor Using Symmetry Structure

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Abstract: We propose a cubic stress tensor sensor that is a rectangular rigid body with symmetrically-located stress sensors and embedded in an elastic body. The sensor measures stress tensor applied to the surface of the elastic body and can distinguish between shear and non-uniformly distributing normal stress. The previously reported tensor sensors had need the information of contact to objects (e.g. robot skin, shoe sole, etc...) [1],[2]. The previously reported tensor sensors had non-uniformly distributing normal stress as shown in Fig. 2. However, they have sensitivity to not only shear stress but also non-uniformly distributing normal stress.

In this paper, we propose a new cube-type tactile sensor embedded in an elastic body. It measures surface stress tensor applied to the body. This cube has normal stress sensors located on each side of the cube as shown in Fig. 1, and can distinguish between non-uniformly distributing normal stress and shear stress.

The stress tensor sensor consists of a cube-shaped rigid body and normal stress sensors placed on each side. It is embedded in a linear elastic body. For simplification, the following discussion focuses on a cross section including the x- and z-axis as shown in Fig. 3.

We assume that the stress tensor sensor is enough small that the surface stress applied to the elastic body consists of the average of stresses \( \sigma_{zz} \), \( \sigma_{yy} \), \( \sigma_{xx} \), and \( \sigma_{xy} \), and gradient of normal stress \( \frac{\partial \sigma_{zz}}{\partial x} \), \( \frac{\partial \sigma_{yy}}{\partial z} \), \( \frac{\partial \sigma_{xx}}{\partial y} \), and \( \frac{\partial \sigma_{xy}}{\partial x} \).

From the symmetry, the relationship of the normal stress sensor’s output in the elastic body \( V_1, V_2, V_3, V_4, V_5, V_6, V_7, \) and \( V_8 \) to input stress tensor applied to the surface of the body are expressed as

\[
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6 \\
V_7 \\
V_8
\end{pmatrix} = \begin{pmatrix}
-1 \\
-1 \\
1 \\
-1 \\
1 \\
-1 \\
1 \\
-1
\end{pmatrix}
\begin{pmatrix}
\sigma_{zz} \\
\sigma_{yy} \\
\sigma_{xx} \\
\sigma_{xy} \\
\sigma_{zz} \\
\sigma_{yy} \\
\sigma_{xx} \\
\sigma_{xy}
\end{pmatrix}
\]

(1)

with constants \( a, b, \ldots, g, \) and \( k \).

As the result, we can reconstruct the stress applied to the elastic body as

\[
\sigma_{zz} = \frac{a}{4L^2 + b^2} \left( \sum_{i=1}^{8} V_i \right) + \frac{b}{a} \sum_{i=1}^{8} \frac{V_i}{b}
\]

(2)

\[
\sigma_{yy} = \frac{a}{4L^2 + b^2} \left( \sum_{i=1}^{8} V_i \right) - \frac{b}{a} \sum_{i=1}^{8} \frac{V_i}{b}
\]

(3)

\[
\sigma_{xx} = \frac{a}{4L^2 + b^2} \left( \sum_{i=1}^{8} V_i \right) - \frac{b}{a} \sum_{i=1}^{8} \frac{V_i}{b}
\]

(4)

In addition, gradient of the normal stress applied to the top surface of the elastic body \( \frac{\partial \sigma_{zz}}{\partial x} \) is reconstructed as

\[
\frac{\partial \sigma_{zz}}{\partial x} = \frac{1}{8(c + d)} \left( V_1 + V_2 + V_3 + V_4 - V_5 - V_6 - V_7 - V_8 \right)
\]

\[
\frac{\partial \sigma_{yy}}{\partial z} = \frac{1}{8(c + d)} \left( V_1 + V_2 + V_3 + V_4 - V_5 - V_6 - V_7 - V_8 \right)
\]

\[
\frac{\partial \sigma_{xx}}{\partial y} = \frac{1}{8(c + d)} \left( V_1 + V_2 + V_3 + V_4 - V_5 - V_6 - V_7 - V_8 \right)
\]

\[
\frac{\partial \sigma_{xy}}{\partial x} = \frac{1}{8(c + d)} \left( V_1 + V_2 + V_3 + V_4 - V_5 - V_6 - V_7 - V_8 \right)
\]

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\[
\frac{\partial \sigma_{zz}}{\partial x} = \frac{h}{4(eg - fh)} \left\{ (-V_1 + V_2 - V_3 + V_4) \right\} \tag{5}
\]

\[
-\frac{E}{h} \left( V'_1 + V'_3 - V'_2 - V'_4 \right) \right\}
\]

3. FEM SIMULATION

To confirm the validity of the model discussed in Section 2, FEM simulations were carried out. A steel rigid cube (Young’s modulus 200 GPa and Poisson’s ratio 0.30) was embedded in a silicone rubber with the Young’s modulus of 0.3 MPa and the Poisson’s ratio of 0.48. The thickness of the silicone rubber was 11 mm and the width was 60 mm. The length of each side of the cube was 4 mm. The cube was placed at the center of the silicone rubber.

The results are shown in Fig. 4. The intensity of the z-component of the normal stress is plotted. Fig. 4 (a) shows the results for the case in which only the shear stress is applied on the surface of the linear elastic body. Fig. 4 (b) is the result for the case in which the normal stress distribution is applied on the surface of the linear elastic body. In the case of Fig. 4 (b), the intensity of the applied normal stress was varied linearly along the x-axis.

In Fig. 4 (a), around the lower left side and upper right side of the rigid square, the elastic body was stretched. In contrast, around the upper left side and lower right side of the rigid square, the compressed region was observed. These results support the symmetrical property of the coefficient matrix discussed in the previous section.

4 EXPERIMENTS

4.1 Prototype

We fabricated a tactile sensing cube (Fig. 5) and embedded it in a soft silicone rubber. The cube consists of an acrylic cube and four capacitive stress sensors placed on the corner of the cube. Each capacitive stress sensor consisted of two copper electrodes and urethane foam. One of the electrodes was connected to GND and the other was connected to the input of a Schmitt inverter so that Schmitt inverter oscillator was formed. The prototype sensor was embedded in a silicone rubber. The frequency of the RC oscillator was determined by the capacitance of the stress sensors. The frequency \( f \) of the oscillator and the distance between the electrodes \( d \) have the following relationship

\[
f = \frac{k}{RC} = \frac{k}{R(e'/S + C'd)} \tag{6}
\]

where \( e' \) is the dielectric constant of the urethane foam, \( S \) is the area of the electrode, \( R \) is the feedback resistance, \( C' \) is the parasitic capacitance and \( k \) is a constant. When normal stress is
applied to the stress sensor, as the Poisson’s ratio of the urethane foam is nearly zero, oscillation frequency variation $\Delta f$, variation $\Delta d$ of $d$, and applied normal stress $\sigma$ have the following relationship

$$\sigma = \frac{E}{d} \Delta d = -\frac{Ek}{\varepsilon S} \frac{\varepsilon}{\varepsilon S} \Delta f,$$ .............................. (7)

where $E$ is the Young’s modulus of the urethane foam. Sensor outputs $V_i$ in Fig. 3 correspond to frequency variation $\Delta f_i$.

According to Eq.(2), (4) and (5), we define $P_{\text{normal}}$ proportional to the surface normal stress $\sigma_{\text{zz}}$, $P_{\text{shear}}$ proportional to the surface shear stress $\sigma_{\text{zx}}$, and $P_{\text{grad}}$ proportional to the gradient of the normal stress, $\frac{\partial \sigma_{\text{zz}}}{\partial x}$, as follows.

$$P_{\text{normal}} = \Delta f_1 - \Delta f_2 - \Delta f_3 - \Delta f_4,$$ .............................. (8)

$$P_{\text{shear}} = \Delta f_1 + \Delta f_2 + \Delta f_3 - \Delta f_4,$$ .............................. (9)

$$P_{\text{grad}} = \Delta f_1 - \Delta f_2 + \Delta f_3 - \Delta f_4,$$ .............................. (10)

Due to the assumption $\sigma_{xx} = 0$, the $x$-direction components corresponding to $V_{5,6,7,8}$ are unnecessary. Fig.8 shows these values calculated from the oscillation frequencies in Fig.7.

In Fig. 8 (a), the horizontal axis represents the displacement of the top steel plate. The vertical axis represents the calculated value of $P_{\text{normal}}$, $P_{\text{shear}}$, and $P_{\text{grad}}$. As shown in Fig. 8 (a), while $P_{\text{normal}}$ increases as the displacement in the $z$-direction increases, $P_{\text{shear}}$ and $P_{\text{grad}}$ don’t seem to have such tendencies though small

4.2 Setup

The cube was embedded in a silicone rubber as shown in Fig.6. Steel plates were placed at the top and bottom of the silicone. The top and bottom steel plates were moved in the $z$- and $x$-direction, respectively. In this experiment, as the boundary condition of the side was free, we assumed $\sigma_{xx} = 0$. The experiments were described with parameters of displacement that we assume proportional to the stress.

4.3 Exeriments and sensor outputs

First, only the top plate was moved in the $z$-direction. Fig. 7 (a) shows the results. The horizontal axis represents the displacement of the top steel plate. The vertical axis represents the oscillating frequencies of each oscillator.

Second, after the silicone was initially strained by moving the top plate in the $z$-direction up to 3 mm, only the bottom plate was moved in the $x$-direction. In this case, it is assumed that shear stress was uniformly applied on the top and bottom surface of the silicone rubber. Fig. 7 (b) shows the results. The horizontal axis represents the displacement of the bottom steel plate.

At last, in order to apply normal stress non-uniformly on the surface of the silicone rubber, a square rod with the width of 5 mm was placed between the top steel plate and the surface of the silicone rubber. The $x$ coordinate of the center of the square rod agreed with that of the capacitive displacement sensor $C_2$. The bottom plate was fixed and only the top plate was moved in the $z$-direction in this case. The results are shown in Fig. 7 (c).

According to Eq.(2), (4) and (5), we define $P_{\text{normal}}$ proportional to the surface normal stress $\sigma_{\text{zz}}$, $P_{\text{shear}}$ proportional to the surface shear stress $\sigma_{\text{zx}}$, and $P_{\text{grad}}$ proportional to the gradient of the normal stress, $\frac{\partial \sigma_{\text{zz}}}{\partial x}$, as follows.

$$P_{\text{normal}} = \Delta f_1 - \Delta f_2 - \Delta f_3 - \Delta f_4,$$ .............................. (8)

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Due to the assumption $\sigma_{xx} = 0$, the $x$-direction components corresponding to $V_{5,6,7,8}$ are unnecessary. Fig.8 shows these values calculated from the oscillation frequencies in Fig.7.

In Fig. 8 (a), the horizontal axis represents the displacement of the top steel plate. The vertical axis represents the calculated value of $P_{\text{normal}}$, $P_{\text{shear}}$ and $P_{\text{grad}}$. As shown in Fig. 8 (a), while $P_{\text{normal}}$ increases as the displacement in the $z$-direction increases, $P_{\text{shear}}$ and $P_{\text{grad}}$ don’t seem to have such tendencies though small
fluctuations can be seen.

In Fig. 8 (b), the horizontal axis represents the displacement of the bottom steel plate. As shown in Fig.8 (b), $P_{\text{shear}}$ increases as the displacement in the $x$-direction increases, while $P_{\text{grad}}$ does not seem to have such tendencies though small fluctuations can be seen. The reason that $P_{\text{normal}}$ decreases is considered that the stationary $z$-direction (compressive) strain decreased by the large shear deformation of the silicone.

In Fig. 7 (c), $P_{\text{grad}}$ increases as the displacement in the $z$-direction increases, as we expected. However, $P_{\text{shear}}$ also shows a similar tendency. The possible explanation for the results is that the point-spread pattern of normal stress distribution does not satisfy the assumption of the symmetry.

5 SUMMARY

We proposed a tactile sensing cube that is embedded in an elastic body and can measure the stress tensor around it. The theory to decompose the shear stress and the normal stress gradient is given. The theory was examined by a prototype sensor. Though the experimental results were not sufficient for fully supporting our proposed principle, the prototype exhibited different responses to the uniformly applied normal stress, the non-uniformly applied normal stress, and the uniformly applied shear stress. We are improving the experimental setup for more precise evaluation. Currently the sensing cube requires wires for signal transmission. In our future design, the electrical power of the sensors and the data signals will be transmitted without complicated long wires [3].

References


![Fig.8 Sensor outputs. (a): Normal stress was uniformly applied. (b): Shear stress was uniformly applied. (c): Linearly varied distribution of normal stress was applied.](image_url)