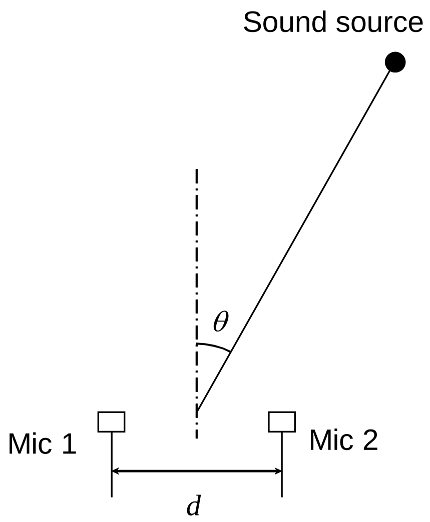

1 The direction of a single point sound source is estimated from the phase difference between the observed acoustic signals by two omnidirectional microphones. You may consider it is a two-dimensional problem and the sound source is sufficiently distant from the microphones. The characteristics of the two microphones are well equalized. The noise is white noise that is uncorrelated with the acoustic signal. You may also suppose there is no correlation between the noises of microphones 1 and 2.

(1) Obtain the theoretical limit of the estimation accuracy on the sound source direction θ, as a function of θ. Assume the signal is a sine wave burst with the rms value of s [V] and the burst duration $T \gg 1/f$, and the microphone spacing is d. Let w [V/$\sqrt{\text{Hz}}$] be the noise density observed by each microphone and c be the speed of sound. You may also assume that the signal strength is much larger than the noise.

(2) Show the above limit when the parameters are given as follows:
Signal: $s = 1$ V, $f = 500$ Hz, $T = 0.1$ s
Noise density: $w = 0.01$ V/$\sqrt{\text{Hz}}$
Sound source direction: around $\theta = 30°$
Speed of sound: $c = 340$ m/s
Distance between microphones 1 and 2: $d = 10$ cm

2 Consider a noise $w(x,y)$ is added to a gray-scale image $s(x-p, y-q)$ of a spherical marker where the marker pattern $s(x,y)$ is known and the marker center position (p,q) is unknown. You are trying to estimate (p,q) from the image of $s(x-p,y-q)$. The noise is random with no correlation among the pixels, and the expected value of $w^2(x,y)$ is a^2. You want to know the limit of the estimation accuracy of (p,q). Explain how you can evaluate the limit.

3 An organism realizes a kind of memory by autonomously changing and retaining the concentration of two chemical components (x_1, x_2) [%] in body fluid. There are three types of sensors that output (y_1, y_2, y_3) reflecting (x_1, x_2) as

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$$

and the sensory system is able to detect each component of (y_1, y_2, y_3) with the accuracy of ± 0.1.

(1) You can find variable transformations $\begin{pmatrix} p \\ q \end{pmatrix} = R_1 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $\begin{pmatrix} r \\ s \\ t \end{pmatrix} = R_2 \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ using orthonormal matrices R_1 and R_2 leading to

$$\begin{pmatrix} r \\ s \\ t \end{pmatrix} = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \\ 0 & 0 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}.$$

Obtain R_1, R_2, α, and β.

(2) Find the variance of each component of (r, s, t) when the variances of the errors of the measured values of (y_1, y_2, y_3) are all equal to σ^2.

(3) Consider the above sensory system is a measurement system that estimates (x_1, x_2) from directly measured values (y_1, y_2, y_3). Describe what properties of the measurement system are derived from R_1, R_2, α, and β.

(4) Obtain the maximum number of bits that this memory system can record in one operation. Assume x_i can take a value in the range of $0 < x_i < 10$, and there is no correlation among the noises added to each component y_i.